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# ATOMIZATION OF A TURBULENT LAYER OF A MIXTURE*

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This article studies the problem of the atomization of a turbulent layer of a mixture formed at the interface of two incompressible media with constant but different densities. It is found that the solution tends toward similarity for long periods of time. The degree of similarity, meanwhile, cannot be determined from dimensional analysis. Instead, it is found during the solution of a boundary-value problem. The degree of similarity is a function of the empirical constants of the model. Similarity solutions are constructed for several parameters, and the dependence of the degree of similarity on the constants of the model is graphed. A formula for the degree of similarity is obtained in an approximation in which the turbulent velocity is constant with respect to the space variable, which the solution for the density of the mixture is expressed through a probability integral. A special case of problem for a uniform medium was examined in [1, 2]. The results of calculations reported there agree with the values obtained in the present study.

1. Formulation of the Problem. A space is filled with two incompressible fluids with the densities $\rho_1^0$ and $\rho_2^0$. The interface passes over a plane. Let a plane turbulent layer of the width $L_0$, consisting of a mixture of both substances, be created at the initial moment of time in the neighborhood of the interface. Such a state can arise, for example, due to the accelerated motion of an interface in the time interval to with the appropriate sign of acceleration. Here, a turbulent layer of the mixture of the width $L_0$ is created during the time $t_0$ and is associated with a certain initial turbulent velocity $v(x, t_0)$. In the absence of turbulence sources, the initial layer of the mixture expands and envelops adjacent fluids. The turbulent energy, determined through the characteristic turbulent velocity, decays in this case and dissipates into heat.

We will use the semiempirical model in [3] to describe the resultant turbulent mixing. This model is based on the balance equation for the kinetic turbulent energy $v^2/2$ and contains three constants. The equations are obtained from the conservation laws for a compressible fluid by means of the substitution $\rho = \bar{\rho} + \rho', u = \bar{u} + u', \ p = \bar{p} + p'$ and corresponding averaging, with the third correlations and the products of the second correlations being discarded. We find from the equation of continuity that $\partial\bar{\rho}/\partial t + \partial\bar{\rho}\tilde{u}/\partial x = 0, \ \tilde{u} = \overline{\rho' u'}/\bar{\rho}.$ Here, we used the incompressibility condition $\bar{u} = 0$.

The equation for the kinetic turbulent energy follows from the continuity law and the momentum conservation law [3, 4]: $(1/2)(\partial\bar{\rho}v^2/\partial t + \tilde{u}\partial\bar{\rho}v^2/\partial x) = -\bar{\nu}\rho v^3/l + (5/6)\bar{\rho}v^2\partial\tilde{u}/\partial x.$ Applying the Prandtl hypothesis $\overline{\rho' u'} = -lv\partial\bar{\rho}/\partial x,$ to the equations, we have

$$\partial\rho/\partial t = \partial(lv\partial\rho/\partial x)/\partial x; \tag{1.1}$$

$$\frac{\partial\rho v^2}{2\partial t} - \frac{lv}{2}\frac{\partial \ln\rho}{\partial x}\frac{\partial\rho v^2}{\partial x} = -\frac{\nu\rho v^3}{l} + \beta\frac{\partial}{\partial x}\left(\rho lv\frac{\partial v^2}{\partial x}\right) + \frac{5\rho v^2}{6}\left[\frac{\partial\ln\rho}{\partial t} - lv\left(\frac{\partial\ln\rho}{\partial x}\right)^2\right], \tag{1.2}$$

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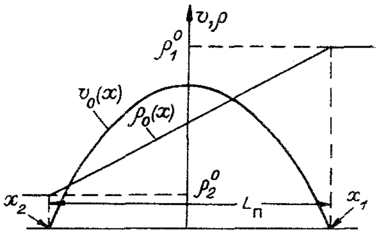


Fig. 1

where $\rho$ is the density of the mixture; $\bar{\rho}$ is replaced by $\rho$. The characteristic turbulence scale $\ell$ is linearly related to the effective width of the mixing region L

$$l = \alpha L \tag{1.3}$$

($\alpha$, $\nu$, $\beta$ are constants of the model, and the form of L is determined below).

In contrast to [3], Eq. (1.2) does not contain a generating term. The first term in the right side generates dissipation of turbulent energy and actually determines the law of decay of the turbulence. The second (diffusional) term, with the coefficient $\beta$, was introduced in [4] to describe spatial spreading of turbulence.

The following problem is posed for system (1.1)-(1.3): determine the solution at t > 0 if at the initial moment (t = 0)

$$v(0,\ x) = v_0(x),\ \rho(0,\ x) = \rho_0(x),\ |x| \leqslant L_0/2 \tag{1.4}$$

($v_0(x)$, $\rho_0(x)$ are functions characterizing the turbulent mixture). The coordinate origin is placed at the middle of the layer (Fig. 1). The boundary conditions on the left and right mixing fronts $x = x_2(t)$ and $x = x_1(t)$ have the form

$$\begin{aligned} x = x_2(t)&:\ v\,[x_2(t),\ t] = 0,\quad \rho_2\,[x_2(t),\ t] = \rho_2^0,\\ x = x_1(t)&:\ v\,[x_1(t),\ t] = 0,\quad \rho_1\,[x_1(t),\ t] = \rho_1^0. \end{aligned} \tag{1.5}$$

The above problem is not self-similar. However, for long periods of time when $t \gg t_0$ and $L \gg L_0$, the initial data should be forgotten and the solution generally tends toward the limiting case.

2. Similarity Solution. System (1.1)-(1.3) allows the similarity transformation

$$x = \tilde{x}_0\lambda\tau^{B/(B+1)},\quad v = \tilde{x}_0^2\zeta\tau^{(B-1)/(B+1)},\quad \rho = \rho_1^0\Delta. \tag{2.1}$$

Here, $\tilde{x}_0$ is a dimensional constant determined by the initial data (1.4); B is an arbitrary dimensionless constant, the similarity index; $\lambda$, $\zeta(\lambda)$, $\Delta(\lambda)$ are the dimensionless representatives of length, velocity, and density; $\tau$ is a new variable related of time by the equation

$$d\tau/dt = l. \tag{2.2}$$

It follows from (2.1) that

$$L = \tilde{x}_0(\lambda_{0.9} - \lambda_{0.1})\tau^{B/(B+1)}, \tag{2.3}$$

where $\lambda_{0.9}$ and $\lambda_{0.1}$ correspond to the coordinates at which the dimensionless density $\delta = (n\Delta - 1)/(n - 1)$ $(n = \rho_1^0/\rho_2^0)$ takes values of 0.9 and 0.1. Here, we have chosen the effective width in accordance with [3]. Otherwise, divergence is seen in the limiting case n = ∞.

We will use the new variables to reduce (1.1)-(1.3) to a system of ordinary differential equations. For this, we substitute (2.1)-(2.3) into (1.1)-(1.3):

$$y((-B/(B+1))\lambda - \zeta') = (y^3 + 2y')\zeta; \tag{2.4}$$

$$\begin{gathered} 2\beta/\zeta(\zeta^2\zeta')' + [(B/(B+1))\lambda + (1+2\beta)\zeta y^2]\zeta' - \\ - \nu\zeta^2/\alpha^2(\lambda_{0.9} - \lambda_{0.1})^2 - [(B-1)/(B+1)]\zeta - (\zeta/3)y^2[(B/(B+1))\lambda + \zeta y^2] = 0. \end{gathered} \tag{2.5}$$

The prime denotes differentiation with respect to $\lambda$. In deriving (2.4) and (2.5), we used the substitution $y^2 = \Delta'/\Delta$, allowing us to reduce the order of the first equation. Boundary conditions (1.5) take the following form in dimensionless variables (2.6)

$$\lambda = \lambda_2:\ \zeta_2 = 0,\ \Delta = 1/n;\ \lambda = \lambda_1:\ \zeta_1 = 0,\ \Delta = 1. \tag{2.6}$$

The solution of the problem for system (2.4)-(2.5) with boundary conditions for the fronts (2.6) is very problematic, especially since the equation has a singularity at points (2.6): the coefficient $\zeta^2$ with the higher derivative vanishes.

However, there is a universal method of solving the above boundary-value problem – numerical integration of the initial equations in partial derivatives (1.1)-(1.3) with initial conditions (1.4). We numerically integrated the initial gasdynamic equations (1)-(5) from [3]. Here, incompressibility was simulated by assigning a sufficiently high initial sonic velocity. We thus establish the fact of an asymptotic solution, which is determined simultaneously. Before going on to discuss the results of the numerical integration, we will make two observations.

Note 1. For a homogeneous medium n = 1, Eq. (2.4) has the trivial solution y = 0, while (2.5) reduces to the form

$$\frac{2\beta}{\zeta}(\zeta^2\zeta')' + \frac{B}{B+1}\lambda\zeta' - \frac{\nu\zeta^2}{\alpha^2(\lambda_{0.9}-\lambda_{0.1})} - \frac{B-1}{B+1}\zeta = 0. \tag{2.7}$$

This case was examined in [1, 2], where it was noted that the similarity index B must be determined during the solution of the boundary-value problem. In fact, the mixing fronts are located symmetrically $\lambda_1 = -\lambda_2 = \lambda_0$. The problem in the new variables $\tilde\lambda$ and $\tilde\zeta$ $(\tilde\lambda = \lambda/\lambda_0,\ \tilde\zeta = \zeta/\lambda_0^2)$ can be reduced to a boundary-value problem on the interval [0, 1] with a symmetry condition at the point $\tilde\lambda = 0$

$$\tilde\zeta' = 0 \tag{2.8}$$

and a completely determined solution at the point $\tilde\lambda = 1$

$$\tilde\zeta = -\frac{B}{(B+1)4\beta}(1-\tilde\lambda) - \frac{1}{(B+1)4\beta}(1-\tilde\lambda)^2 + \ldots \tag{2.9}$$

In the variables $\tilde\lambda$ and $\tilde\zeta$, we write Eq. (2.7) in the form

$$(2\beta/\tilde\zeta)(\tilde\zeta^2\tilde\zeta')' + (B/(B+1))\tilde\lambda\tilde\zeta' - \nu\tilde\zeta^2/(4\alpha^2\lambda_{0,1}^2) - ((B-1)/(B+1))\tilde\zeta = 0.$$

We find the solution by numerical integration of the last equation. Starting from the point $\tilde\lambda = 1$ in expansion (2.9) and integrating to the point $\tilde\lambda = 1$, we select a value of the similarity parameter such that the condition at the center of symmetry (2.8) is satisfied.

Note 2. The degree of similarity B is a function of the model constants $\beta$, $\alpha$, and $\nu$. The last two constants appear in the form of the ratio $\nu/\alpha^2$. This fact was not noted in [1, 2], where the coefficients of Eq. (3.6) depend on the parameters $\alpha$ and $\nu$. Replacement of the sought solution in [1, 2] by the new solution $\tilde\Phi$ $(\tilde\Phi = \alpha^2\Phi)$ leads to an equation with one coefficient proportional to $\nu/\alpha^2$ (in [1, 2], $\nu$ = c, $\beta$ = 0.25).

3. Results of the Calculations and Discussion. Figures 2-6 show the results of numerical integration of the initial equations in partial derivatives. The solution was obtained in the program TURINB by the method in [3]. As the initial data we took the following values of $v_0(x)$ and $\rho_0(x)$: $v_0(x) = v_0$ – constant, $\rho(0, x) = \rho_2^0 + (\rho_1^0 - \rho_2^0)(x/L_0 + 0.5)$, $|x| \leqslant L_0/2$.

We examined the dependence of the solution on the initial parameters $\beta$, $\nu/\alpha^2$, and n. It was established that the values of $\beta$ and n have a slight effect on the degree of similarity B. This follows from Fig. 2, where the dependence of the degree B on the ratio $\nu/\alpha^2$ is shown at n = 1. The points show results of numerical integration with $\beta$ = 0.25, while the curve shows the approximate relation given by Eq. (4.6). We also numerically determined the solution at $\beta$ = 0.75. The difference in the values of B is less than 1% and cannot be discerned in the figures.

Figure 3 shows results of calculations with a fixed value of the coefficient $\beta$ ($\beta$ = 0.25) and n = 3; 10; 20 (points 1-3) in relation to $\nu/\alpha^2$.

Profiles of the dimensionless velocity $\zeta$ and density $\delta$ are shown by Figs. 4 and 5, respectively. The structure of the solution in the neighborhood of the front follows from the expansion $\zeta = [B\lambda_i/(4(B+1)\beta)](\lambda_i - \lambda) + \ldots$, $y = D_i(\lambda_i - \lambda)^{(4\beta-1)/2} + \ldots$ $(i = 1, 2)$ ($D_i$ are constants). The expansion was obtained with finite values of $\lambda_1$ and $\lambda_2$. The mixing front is absent only in the approximate solution of Part 4, where the turbulent velocity $\zeta$ is assumed to be independent of the space coordinate. The expansion for the function y is nonanalytical

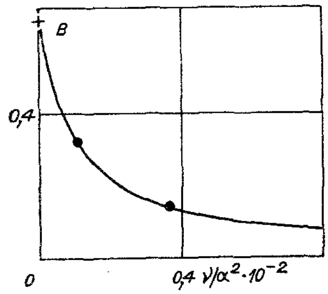


Fig. 2

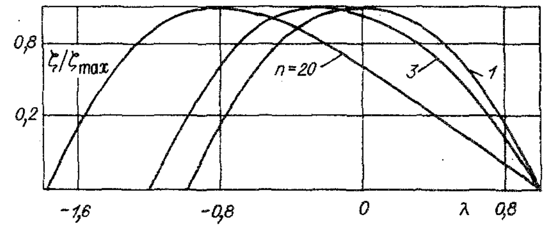


Fig. 4

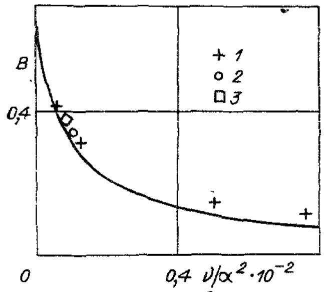


Fig. 3

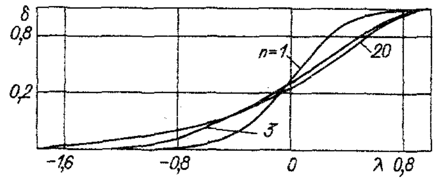


Fig. 5

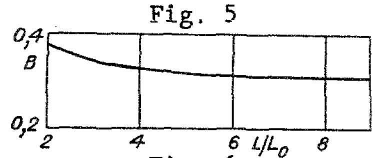


Fig. 6

in character. In this sense, the value $\beta = 0.25$ is critical. For this value, there is an expansion in the form of a series in integral powers, while the function y takes a finite value at the points $\lambda_1$ and $\lambda_2$.

The profile of velocity $\zeta$ is symmetrical at n = 1, but the symmetry disappears if n > 1: the maximum value of velocity shifts in the direction of the lighter substance with an increase in n. The mixing front also moves in this direction. Figure 6 shows the arrival of the problem at a similarity regime with initial data (1.4) and $\nu/\alpha^2 = 12.5$. The initially nonsimilar velocity and density profiles also approach the similarity profiles in the limit, the latter being depicted in Figs. 4 and 5.

4. Approximate Solution. Analyzing the profiles of the solutions obtained (see Fig. 4), we see that turbulent velocity is bell-shaped in character in the mixing region. Thus, as in [5], we can construct an approximate solution by replacing turbulent velocity in the mixing region with a constant. To do this, we average Eq. (1.2) over the mixing region:

$$\frac{d\bar{v}^2}{2d\bar{\tau}} = -\frac{k\bar{v}^2}{\bar{\tau}}, \quad k = 0.25 + \frac{\nu}{16\eta_0^2\alpha^2} + \frac{2}{3\pi}\left(\frac{n-1}{n+1}\right)^2; \tag{4.1}$$

$$d\bar{\tau} = l\bar{v}dt; \tag{4.2}$$

$$\delta = 0.5(1 + \Phi(x/(2\bar{\tau}^{0.5}))); \tag{4.3}$$

$$L = 4\eta_0\bar{\tau}^{0.5}, \quad \eta_0 = 0.906. \tag{4.4}$$

Here, we have also written out the solution for L and $\delta$; $\Phi$ is the probability integral: $\Phi(\eta) = \frac{2}{\pi^{0.5}}\int\limits_0^\eta \exp(-\eta^2)\,d\eta$. We used the solution for density (4.3) in deriving (4.1). Equation (1.2) was averaged by integration over the mixing region $|x| \le L/2$. First, in (1.2), we replaced the time t by the variable $\bar{\tau}$, and after integration over the mixing region the corresponding integrals were replaced by approximate expressions

$$\int\limits_{|x|\le\frac{L}{2}} \frac{\partial\rho v^2}{\partial\bar{\tau}}\,dx \simeq \frac{\partial(\bar{v}^2M)}{\partial\bar{\tau}},$$

$$\int\limits_{|x|\le\frac{L}{2}} \frac{\partial\rho}{\partial x}\frac{\partial v^2}{\partial x}\,dx \simeq \left.\frac{\partial\rho}{\partial x}\right|_{x=0} \int\limits_{|x|\le\frac{L}{2}} \frac{\partial v^2}{\partial x}\,dx = 0,$$

$$\int\limits_{|x|\leqslant\frac{L}{2}} v^2\left(\frac{\partial \ln\rho}{\partial x}\right)^2 \rho\, dx \simeq \left(\frac{\partial \ln\rho}{\partial x}\right)^2_{x=0} \bar{v}^2 M,$$

$$M = \int\limits_{|x|\leqslant\frac{L}{2}} \rho\, dx = \frac{\rho_1^0+\rho_2^0}{2} L, \quad \left.\frac{\partial \ln\rho}{\partial x}\right|_{x=0} = \frac{n-1}{(n+1)\,(\pi\tau)^{0,5}}.$$

Equations (4.1)-(4.4) are integrated and the solution is written in the form

$$L = At^{1/(1+2k)}, \quad \bar{v} = \left[A/(8\alpha\eta_0^2(1+2k))\right] t^{-2k/(1+2k)}. \tag{4.5}$$

Here, A is a constant determined by the initial data (1.4). Comparing Eqs. (4.5) and (2.3), we find the explicit expression

$$B = 1/\left[1.5 + \nu/(8\eta_0^2\alpha^2) + (1/3\pi)\,((n-1)/(n+1))^2\right], \tag{4.6}$$

which is also valid at n = 1. The relation for this case is shown by the curve in Fig. 2. It should be noted that Eq. (4.6) can also be obtained by constructing the approximate solution for system (2.4)-(2.5). This was done in the appendix. Evaluation (4.6) is quite satisfactory for the general case as well. The line in Fig. 3 shows the degree B from Eq. (4.6) with n = 3. The results of the numerical integration are compared with approximate formula (4.6) in Figs. 2 and 3, illustrating the good accuracy of the approximate formula. The numerical results [1, 2] agree with the curve in Fig. 2.

In comparing B, it should be kept in mind that the value of the constant $\alpha$ depends on the width used in Eq. (1.3). Thus, if the total width $L_t$ figures in the model, then the constants $\alpha_t$ and $\alpha$ will be connected by the relation $\alpha_t L_t = \alpha L$.

The problem is simplified only slightly for an arbitrary value of n at $\nu = 0$, and the degree B also remains indeterminate and is found during the solution of the problem. The only exception is the value of the parameter n equal to 1, when B = 2/3. The overall result obtained at $n \neq 1$ follows from approximate expression (4.5) for B. It also follows from the form of Eq. (1.2): the law of conservation of initial turbulent energy does not hold in the case of arbitrary n.

Appendix. We will construct an approximate solution of system (2.4)-(2.5). To do this, we ignore the term $\zeta'$ in (2.4) and we replace the function $\zeta$ by the constant $\zeta_0$. We determine the latter by approximate integration of Eq. (2.5). As a result

$$-B/((B+1)\zeta_0)y\lambda = y^3 + 2y'; \tag{A.1}$$

$$(1+B)\,\zeta_0 = (1-1.5B)/\left(\nu/(4\alpha^2\lambda_{0.1}^2) + y_0^4/\zeta\right). \tag{A.2}$$

The last relation was obtained as follows. Equation (2.5) was multiplied by $\zeta$ and we took the integral over the region $[-\lambda_{0,1}, \lambda_{0,1}]$, from both sides. Here, we used the approximate equalities

$$\int\limits_{-\lambda_{0.1}}^{\lambda_{0.1}} \left[\frac{B}{B+1}\lambda + (1+2\beta)\,\zeta y^2\right]\zeta\zeta'\, d\lambda \simeq -\frac{B}{B+1}\lambda_{0.1}\zeta_0^2,$$

$$\int\limits_{-\lambda_{0.1}}^{\lambda_{0.1}} \zeta^2 y^2\left(\frac{B}{B+1}\lambda + \zeta y^2\right) d\lambda \simeq 2y^4(0)\,\zeta_0^3\lambda_{0.1}.$$

Differential equation (A.1) for the function y is the Bernoulli equation. It is integrated, and the solution is represented in the form $\Delta = 1/y^2(0) + [((1+B)/(2B))\zeta_0\pi]^{0.5}\Phi[\lambda B^{0.5}/((2(B+1)\zeta_0)^{0.5})]$.

Satisfying boundary conditions (2.6), we have

$$y_0^2 = 2n/(n+1), \quad [2(1+B)\,\zeta_0\pi]^{0.5} = (n-1)/n. \tag{A.3}$$

We find from the condition $\delta(\lambda_{0.1}) = 0.1$ that

$$\lambda_{0,1} = \eta_0(n-1)/(\pi^{0.5}n). \tag{A.4}$$

We can use Eqs. (A.2)-(A.4) to determine the similarity index B. The expression for the index coincides identically with Eq. (4.6).

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# NUMERICAL STUDY OF SWIRLING ONE- AND TWO-PHASE TURBULENT FLOWS IN A CYLINDRICAL CHANNEL

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UDC 532.517.4

Turbulent swirling flows are widely used to intensify heat and mass transfer processes in different types of processing units. Examples of the latter are plasma-chemical reactors plasmatrons, combustion chambers, scrubbers, etc. To make these units more efficient, it is necessary to make a detailed study of the hydrodynamics in swirling flows.

It is known that swirling flows are characterized by highly curved streamlines and the development of recirculation zones. The location and dimensions of these zones depend to a considerable extent on the intensity of swirling and the configuration of the boundaries of the flow. The dimensions of the recirculation zones also depend on the "charging" of the flow with particles in the case of dispersed-gas flows. The study of vortical flows with a disperse phase is complicated by the need to allow for dynamic interaction of the phases. This, together with the problem of modeling the turbulence, makes it more difficult to numerically study such flows. The theoretical and experimental investigation of swirling flows was given great impetus in [1-3].

1. Swirling Turbulent One-Phase Flows. The large amount of interest in intensive swirling flows – the main type of turbulent flow – requires the use of fairly flexible turbulence models. The study [4] presented the results of calculations of axisymmetric swirling turbulent jets using the Prandtl mixing length model. The results agreed well with experimental findings. In [5, 6] an attempt was made to use the standard $k-\epsilon$ model of turbulence to numerically study swirling flows ($k$ is the kinetic energy of the pulsating motion and $\epsilon$ is the rate of dissipation of pulsative energy). This model has proven to be useful in calculations of simple shear flows. However, use of the standard $k-\epsilon$ model in the case of fairly intensive swirling has led to a significant deviation from the experimental results. The authors of [6] explain this discrepancy by citing the anisotropy of eddy viscosity, although the standard turbulence model they used does not even take into account the expressions for the fluctuation moments which appear due to swirling and make a description possible within the framework of an isotropic model. It was noted in [7] that one way of further improving turbulence models for swirling flows is modifying the $k-\epsilon$ model in different ways.

In [8-12], corrections were proposed for the traditional two-parameter model. As noted in [12], all of the modifications proposed earlier for the $k-\epsilon$ model proved unsuitable for calculating bounded swirling flows. The approach taken by the authors of [12] consisted of selecting optimum values of the empirical constants of the energy–dissipation model to study

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